

November 1995



UMDEPP 96-46

On Aspects and Implications of the New Covariant 4D, $N = 1$ Green-Schwarz σ -model Action¹

S. James Gates, Jr.
Department of Physics
University of Maryland
College Park, MD 20742-4111 USA

`gates@umdhep.umd.edu`

ABSTRACT

Utilizing $(2,0)$ superfields, we write a (supersymmetry)² action and partially relate it to the new formulation of the Green-Schwarz action given by Berkovits and Siegel. Recent results derived from this new formulation are discussed within the context of some prior proposals in the literature. Among these, we note that 4D, $N = 1$ β FFC superspace geometry with a composite connection for \mathcal{R} -symmetry has now been confirmed as the only presently known limit of 4D, $N = 1$ heterotic string theory that is derivable in a completely rigorous manner.

¹Supported in part by National Science Foundation Grant PHY-91-19746
 and by NATO Grant CRG-93-0789

1 Introduction

Near the very beginning of our study of two-dimensional locally supersymmetric field theories that are relevant to superstrings [1], we suggested the radical notion that the ultimate formulation of superstring actions must be a bizarre hybrid of the Green-Schwarz (GS) and Neveu-Schwarz-Ramond (NSR) actions. The reason for making this proposal was in answer to a simple but deeply troubling question, “Why do the two seemingly different formulations (NSR vs. GS) both exist?” It seemed to us that the simplest resolution to this puzzle was the idea that the two different formulations were really just two different ways (possibly even different gauge choices) of looking at a meta-string formulation that contained elements of both. Hence, our seemingly fanciful suggestion. Our original proposal can be put in the form of the $(\text{SUSY})^2$ principle:

The ultimate covariant formulation of superstring theory must involve a set of variables that describe a map from a world-sheet supermanifold into a space-time supermanifold.

The standard GS formulation may be thought of as a map from a 2d world-sheet into a supermanifold and the NSR formulation may be thought of as a map from a super 2d manifold into a bosonic manifold. So our suggestion was just the next logical progression. In a series of paper [2], we have explored various ways in which the realization of the $(\text{SUSY})^2$ principle might lead to an improved version of superparticle and superstring theory. As well, numbers of other authors [3] have investigated variations on this often-rediscovered “crazy idea” [4].

A more recent development program [5], has occurred that yields a result that comes tantalizingly close to the successful realization of the $(\text{SUSY})^2$ principle. This is most evident in a recent paper [6]. A manifest, classical $(\text{SUSY})^2$ realization is not present. However, at the quantum level, this model does apparently provide a realization. In this light, it is clearly an important object for our study. In the present work, we will discuss aspects of this new σ -model and show how it can be embedded into a classical $(\text{SUSY})^2$ model. We will also see how this new model settles a number of issues that were raised in previous investigations of 4D, $N = 1$ Green-Schwarz non-linear σ -models [7]. Perhaps the most important along these lines is to note that [6] provides an independent and rigorous derivation of the 4D, $N = 1$ supergravity limit. Namely, it is found that the 4D, $N = 1$ supergravity theory emerging from the heterotic string is the “old minimal” supergravity theory “entangled” with a tensor multiplet that acts as the composite connection for \mathcal{R} -symmetry exactly as described in reference [7].

2 Review of Local (2,0) Superspace Supergravity Geometry

Some time ago, the geometry of (2,0) supergravity was developed [8]. At that time, however, the formulation did not take advantage of the fact that in conformal theories, the auxiliary field G_- couples to matter exactly like a U(1) gauge field. To make this obvious, it is convenient to modify the (2,0) supergravity covariant derivative in [8] by introducing a world-sheet U(1) generator $\hat{\mathcal{Y}}$ and redefine $\nabla_A \rightarrow \nabla_A + \delta_A{}^B G_{-B} \hat{\mathcal{Y}}$. The resulting (2,0) supergravity covariant derivative satisfies,

$$\begin{aligned} [\nabla_+, \nabla_+] &= 0 \quad , \quad [\nabla_+, \bar{\nabla}_+] = i\nabla_{\#} \quad , \\ [\nabla_+, \nabla_{\#}] &= 0 \quad , \quad [\nabla_+, \nabla_-] = i\bar{\Sigma}^+ (\mathcal{M} + i\hat{\mathcal{Y}}) \quad , \\ [\nabla_{\#}, \nabla_-] &= \Sigma^+ \nabla_+ + \bar{\Sigma}^+ \bar{\nabla}_+ + \mathcal{R}\mathcal{M} + \mathcal{F}\hat{\mathcal{Y}} \quad , \end{aligned} \quad (2.1)$$

where Σ^+ , \mathcal{R} and \mathcal{F} are superfields such that

$$\begin{aligned} \Sigma^+ &\equiv [\mathcal{D}_{\#}\chi_- - \mathcal{D}_-\chi_{\#} - c_{\#}{}^c \chi_c^+] \quad , \\ \mathcal{R} &\equiv r(e, \omega(e, \chi)) + i[\chi_{\#}^+(\bar{\Sigma}^+)| + \bar{\chi}_{\#}^+(\Sigma^+)|] \quad , \\ \mathcal{F} &\equiv \mathcal{D}_{\#}A_- - \mathcal{D}_-A_{\#} - c_{\#}{}^c A_c - [\chi_{\#}^+(\bar{\Sigma}^+)| - \bar{\chi}_{\#}^+(\Sigma^+)|] \quad , \end{aligned} \quad (2.2)$$

and $r(e, \omega(e, \chi))$ denotes the world sheet curvature. The component field content is just $(e_a{}^m, \chi_a^+, A_a)$. The field strength superfields satisfy

$$\bar{\nabla}_+ \Sigma^+ = 0 \quad , \quad \nabla_+ \Sigma^+ = \mathcal{R} + i\mathcal{F} \quad . \quad (2.3)$$

The final result that is required to derive component results from (2,0) superspace results is to note the “density projection” formulae;

$$\begin{aligned} \int d^2\sigma d^2\zeta^{\#} E^{-1} \mathcal{L} &\equiv \frac{1}{2} \int d^2\sigma d\zeta^+ \mathcal{E}^{-1} ((\nabla_+ - i2\bar{\chi}_{\#}^+) \bar{\nabla}_+ \mathcal{L})| \\ &+ \frac{1}{2} \int d^2\sigma d\bar{\zeta}^+ \bar{\mathcal{E}}^{-1} ((\bar{\nabla}_+ - i2\chi_{\#}^+) \nabla_+ \mathcal{L})| \end{aligned} \quad (2.4)$$

valid for any (2,0) superfield \mathcal{L} and as well

$$\int d^2\sigma d\zeta^+ \mathcal{E}^{-1} \mathcal{L}_{-c} = \int d^2\sigma e^{-1} ((\nabla_+ - i2\bar{\chi}_{\#}^+) \mathcal{L}_{-c})| \quad , \quad (2.5)$$

valid for any chiral superfield \mathcal{L}_{-c} .

One other interesting feature is the form of the (2,0) world sheet scale (Howe-Tucker) transformation laws of the covariant derivative. These take the forms

$$\begin{aligned}
\delta_L \nabla_+ &= \tfrac{1}{2} L \nabla_+ - (\nabla_+ f) \mathcal{M} - (\nabla_+ g) \hat{\mathcal{Y}} \quad , \\
\delta_L \nabla_{\neq} &= \tfrac{1}{2} [L + \bar{L}] \nabla_{\neq} - i \tfrac{1}{2} [\bar{\nabla}_+ (L - \bar{f} - i\bar{g})] \nabla_+ \\
&\quad - i \tfrac{1}{2} [\nabla_+ (\bar{L} - f + ig)] \bar{\nabla}_+ \\
&\quad + i [\bar{\nabla}_+ \nabla_+ f + \nabla_+ \bar{\nabla}_+ \bar{f}] \mathcal{M} \\
&\quad + i [\bar{\nabla}_+ \nabla_+ g + \nabla_+ \bar{\nabla}_+ \bar{g}] \hat{\mathcal{Y}} \quad , \\
\delta_L \nabla_{=} &= \tfrac{1}{2} [L + \bar{L}] \nabla_{=} - (\nabla_{=} F) \mathcal{M} - (\nabla_{=} G) \hat{\mathcal{Y}} \quad ,
\end{aligned} \tag{2.6}$$

where the parameter superfields L , f , g , F and G are all expressed in terms of a chiral superfield Λ

$$\begin{aligned}
L &\equiv \tfrac{1}{2} (\Lambda + \bar{\Lambda}) \quad , \quad f \equiv -\tfrac{1}{2} \Lambda \quad , \quad F \equiv \tfrac{1}{2} (\Lambda + \bar{\Lambda}) \quad , \\
g &\equiv -i \tfrac{1}{2} (2\Lambda + \bar{\Lambda}) \quad , \quad G \equiv 0 \quad .
\end{aligned} \tag{2.7}$$

These imply the following transformations of the field strength Σ^+

$$\delta_L \Sigma^+ = \tfrac{3}{2} L \Sigma^+ + i (\nabla_{=} \nabla_+ \Lambda) , \tag{2.8}$$

and we note that the transformation of the other two field strength superfields follows from applying δ_L to the latter result in (2.3).

In closing this section, we wish to return to the issue of uniqueness of (2,0) supergravity. As we pointed out in the introduction to this section, our first description of (2,0) supergravity did not include a gauged U(1). Since from the view of string theory this U(1) appears significant, it is certainly reasonable to ask if there are any alternatives? The reason for asking this question is that the form of the (2,0) supergeometry is ultimately responsible for the type of space-time conformal compensator (see the discussion below) that is utilized. This question of uniqueness is also related to the question of whether there are more (2,0) scalar multiplets in addition to the chiral one? The answer to both of these questions is yes. There is another way to take our initial construction of (2,0) supergravity and gauge U(1). All that needs to be done is to take the covariant derivative in [8] and change it according to $\nabla_A \rightarrow \nabla_A + \Gamma_A \hat{\mathcal{Y}}$ introducing a (2,0) matter vector multiplet (Γ_A) that is independent of (2,0) supergravity. The most important feature of this alternative description is that this theory possess a U(1) covariant superfield $G_{=}$ that is the lowest component of the supergravity field strength multiplet. Under this circumstance it can be shown

that the Fradkin-Tseytlin term is given by

$$\mathcal{S}_{FT} = \int d^2\sigma d^2\zeta^\sharp E^{-1} \Phi G_- , \quad (2.9)$$

where Φ is an arbitrary function of world-sheet scalar superfields. This observation might open the way to alternative formulations of the 4D, $N = 1$ supergravity theory (non-minimal, new minimal) derived from heterotic superstrings.

With regard to the existence of more (2,0) scalar multiplets, we have an answer that is derivable from some quite recent work on WZNW terms [9]. There it was shown that there exist a (2,2) scalar multiplet called the non-minimal scalar multiplet. This representation is distinct from chiral multiplets and possesses a (2,0) truncation and is thus a candidate to appear in a different world sheet action. The superfield description of this (2,0) multiplet requires two superfields, denoted by Y and P_- , that satisfy the constraint $\bar{\nabla}_+ \nabla_- Y = \bar{\nabla}_+ P_-$. The free action for the multiplet is just

$$\int d^2\sigma d^2\zeta^\sharp E^{-1} [iY\bar{P}_- + \text{h.c.}] \quad . \quad (2.10)$$

3 Manifest (SUSY)² vs. the New Green-Schwarz Action

The (SUSY)² principle implies that the object of primary interest is $\mathcal{Z}^{\underline{M}}$ that maps from (2,0) superspace into 4D, $N = 1$ superspace. One choice to represent this map is $\mathcal{Z}^{\underline{M}} = (\mathcal{Z}^\mu, \mathcal{Z}^{\dot{\mu}}, \mathcal{Z}^{\mu\dot{\mu}})$ where $\mathcal{Z}^{\underline{M}}$ is a (2,0) superfield, i.e. $\bar{\nabla}_+ \mathcal{Z}^{\underline{M}} = 0$. Since necessarily $\mathcal{Z}^{\underline{M}}$ is complex we may write

$$\mathcal{Z}^{\mu\dot{\mu}} \equiv X^{\mu\dot{\mu}} + iY^{\mu\dot{\mu}} \quad , \quad \mathcal{Z}^\mu \equiv \frac{1}{2}[\Theta^\mu + \Delta^\mu] \quad , \quad \mathcal{Z}^{\dot{\mu}} \equiv \frac{1}{2}[\bar{\Theta}^{\dot{\mu}} - \bar{\Delta}^{\dot{\mu}}] \quad , \quad (3.1)$$

where $X^{\mu\dot{\mu}}$ and $Y^{\mu\dot{\mu}}$ are real. We identify the usual 4D space-time superstring coordinates by

$$\frac{1}{2}[\mathcal{Z}^{\mu\dot{\mu}} + (\mathcal{Z}^{\mu\dot{\mu}})^*](\zeta^+, \bar{\zeta}^+, \sigma, \tau) = \begin{pmatrix} X^0 + X^3 & X^1 - iX^2 \\ X^1 + iX^2 & X^0 - X^3 \end{pmatrix} \quad . \quad (3.2)$$

$$[\mathcal{Z}^\mu + (\mathcal{Z}^\mu)^*](\zeta^+, \bar{\zeta}^+, \sigma, \tau) = \Theta^\mu(\sigma, \tau) \quad . \quad (3.3)$$

As readily seen, manifest (2,0) supersymmetry has forced us to introduce a sort of mirror superspace with coordinates $(\Delta^\mu, Y^{\mu\dot{\mu}})$. We can turn this to our advantage.

It is well known that there are three bases in which to represent Salam-Strathdee superspace; (a.) vector basis, (b.) chiral basis and (c.) anti-chiral basis. By placing constraints on the mirror supercoordinates we seem able to represent each of these. For example, the vector basis seems related to the choice $Y^{\mu\dot{\mu}} = \Delta^\mu = 0$ and the chiral basis seems related to $Y^{\mu\dot{\mu}} = \Theta^\mu \bar{\Theta}^{\dot{\mu}}$ and $\mathcal{Z}^\mu = 0$ (note that \mathcal{Z}^μ is not the conjugate of $(\mathcal{Z}^\mu)^*$). The chiral basis provides a minimal way in which to describe the superspace.

As the 2D world-sheet supergeometry is characterized by the (2,0) supergravity covariant, ∇_A , in order to describe the 4D, $N = 1$ space-time supergeometry, we introduce a vielbein $E_{\underline{M}}^{\underline{A}}$ that is a function of the coordinates $\mathcal{Z}^{\underline{M}}$ and $\bar{\mathcal{Z}}^{\underline{M}}$. The quantities $\Pi_A^{\underline{A}} \equiv (\Pi_+^{\underline{A}}, \Pi_\mp^{\underline{A}}, \Pi_-^{\underline{A}})$ denote space-time supercovariant “normals” that are defined by,

$$\Pi_+^{\underline{A}} = (\nabla_+ \mathcal{Z}^{\underline{M}}) E_{\underline{M}}^{\underline{A}} \quad , \quad \Pi_\mp^{\underline{A}} = (\nabla_\mp \mathcal{Z}^{\underline{M}}) E_{\underline{M}}^{\underline{A}} \quad , \quad \Pi_-^{\underline{A}} = (\nabla_- \mathcal{Z}^{\underline{M}}) E_{\underline{M}}^{\underline{A}} \quad , \quad (3.4)$$

and satisfy

$$F_{AB}^{\underline{C}} \equiv \nabla_A \Pi_B^{\underline{C}} - (-)^{AB} \nabla_B \Pi_A^{\underline{C}} - T_{AB}^{\underline{C}} \Pi_C^{\underline{C}} - (-)^{AB} \Pi_A^{\underline{A}} \Pi_B^{\underline{B}} T_{\underline{A}\underline{B}}^{\underline{C}} = 0. \quad (3.5)$$

In other words, if $\Pi_A^{\underline{A}}$ is regarded as a world-sheet gauge field, it has a vanishing field strength where $T_{\underline{A}\underline{B}}^{\underline{C}}$ acts as the structure constants for the gauge group. Alternately, we may regard $\Pi_A^{\underline{A}}$ as a linear mapping operator that relates vectors and covectors on the superworld-sheet to those over the 4D, $N = 1$ super space-time via $e_A = \Pi_A^{\underline{A}} E_{\underline{A}}$ and $d\omega^{\underline{A}} = d\omega^A \Pi_A^{\underline{A}}$. Although (3.5) is a classical equation, it is interesting to conjecture that its expectation value in a quantized theory is related to anomalies and critical dimensions.

Now having completed our definitions, we note that the remaining component fields in $\mathcal{Z}^{\underline{M}}$ (complex bosonic twistor fields and complex NSR fermions) may be defined covariantly with respect to both world-sheet and space-time manifolds through the equation

$$|\Pi_+^{\underline{A}}| \equiv (S_+^\alpha, \tilde{S}_+^{\dot{\alpha}}, \psi_+^a) \quad . \quad (3.6)$$

For an action to describe the dynamics of $\mathcal{Z}^{\underline{M}}$, we take our motivation from the symmetries (both classical and quantum) of the action of reference [6] and write

$$\begin{aligned} \mathcal{S} = & \left\{ \int d^2\sigma d^2\zeta^\mp E^{-1} [\bar{\mathcal{Z}}^{\underline{M}} E_{\underline{M}}^{\underline{A}} \Pi_-^{\underline{B}} t_{\underline{A}\underline{B}}^{(0)} + \Pi_+^{\underline{A}} \Pi_+^{\underline{B}} \Lambda_{\underline{A}\underline{B}}] + \text{h.c.} \right\} \\ & + \left\{ \int d^2\sigma d^2\zeta^\mp E^{-1} [\Pi_+^{\underline{A}} \Lambda_{\underline{A}\underline{B}} t_{\underline{A}\underline{B}}^{(1)} + \Pi_\mp^{\underline{A}} \Lambda_{\underline{A}\underline{B}} t_{\underline{A}\underline{B}}^{(2)} + \Pi_-^{\underline{A}} \Lambda_{\underline{A}\underline{B}} t_{\underline{A}\underline{B}}^{(3)}] + \text{h.c.} \right\} \\ & + \left\{ \int d^2\sigma d\zeta^+ \mathcal{E}^{-1} [\Sigma^+ \Phi(\mathcal{Z})] + \text{h.c.} \right\} \quad , \end{aligned} \quad (3.7)$$

where the quantities $t_{\underline{A}\underline{B}}^{(i)}$ are a set of constant tensors. One parametrization of these is

$$t_{\underline{A}\underline{B}}^{(i)} = \begin{pmatrix} k_1^{(i)} C_{\alpha\beta} & 0 & 0 \\ 0 & k_2^{(i)} C_{\dot{\alpha}\dot{\beta}} & 0 \\ 0 & 0 & k_3^{(i)} C_{\alpha\beta} C_{\dot{\alpha}\dot{\beta}} \end{pmatrix} . \quad (3.8)$$

For $k_1^{(0)} = k_2^{(0)} = 0$, $k_3^{(0)}$ nonvanishing, the first term produces the standard nonlinear σ -model with torsion for $|\mathcal{Z}^{\underline{M}}|$ [8, 10]. For $k_3^{(1)}$ nonvanishing, variation with respect to $\Lambda_{-}^{\underline{b}}$ imposes the superfield equation $\Pi_{+}^{\underline{b}} = 0$. In the work of reference [11], the analog of this condition plays a critical role in eliminating would-be NSF fermions. Finally, for $k_1^{(1)}$ and $k_2^{(3)}$ nonvanishing, a simple definition of propagators for the Grassmann coordinates (Θ^α) is possible. We don't completely understand the role of $t_{\underline{A}\underline{B}}^{(2)}$, it seems related to the choice of basis (vector, chiral, anti-chiral).

No explicit factors of α' appear in our action. The reason for this is that we can relegate all such factors to the zero modes of $\mathcal{Z}^{\underline{M}}$. By this we mean a mode expansion takes the forms

$$X^{\mu\dot{\mu}}(\sigma, \tau = \tau_0) = (4\pi\alpha')^{-\frac{1}{2}} x^{\mu\dot{\mu}} + \dots , \quad \Theta^\mu(\sigma, \tau = \tau_0) = (4\pi\alpha')^{-\frac{1}{4}} \theta^\mu + \dots . \quad (3.9)$$

where \dots indicates higher mode terms. Using this convention, α' never appears anywhere else in the formalism and all the two dimensional fields possess natural units of engineering dimensions (i.e., 2d bosons = 0, 2d fermion = $\frac{1}{2}$). Clearly, the component level evaluation of the action is an important next step. Since this promises to be quite intricate, we will carry out this analysis in a future work. In closing this section, we wish there to be no misunderstanding. We are not presently claiming that the action of (3.7) is the same as the σ -model of Berkovits and Siegel. Instead we propose it as the starting point in trying to construct a classical manifestly (SUSY)² model that matches many properties of their construction.

4 Will the Real 4D, N = 1 Supergravity Limit of Heterotic String Theory Please Stand up?

There are many reasons why a complete manifest realization of superstring theory is desirable. Presently, many misunderstandings exist due to such a complicated theory being formulated in such an incomplete manner. An example of this arose several years ago [12, 13] regarding the pure 4D, N = 1 supergravity limit of heterotic superstring theory. At first [12] it was argued that the “new minimal” off-shell version

of 4D, $N = 1$ supergravity must necessarily occur at this limit. It was later [13] noted that such a proposal was inconsistent with the superspins implied by independent superstring theory arguments. In [7], an analysis was performed to see how these two competing claims could occur and, remarkably enough, it was shown that there existed an ambiguity in the interpretation of the results of [12]. Those results are equivalent to a derivation of the form of the graded commutator algebra of the superspace supergravity covariant derivative. In [7] it was shown that the superspace supergravity covariant derivative thus derived could be expressed as either the “new minimal” off-shell version of 4D, $N = 1$ supergravity or as the “old minimal” off-shell version of 4D, $N = 1$ supergravity “entangled” with a tensor multiplet that is also used as a composite connection for \mathcal{R} -symmetry. This last interpretation may seem counter intuitive and unnatural but it provided the only logical way to reconcile the different claims [12, 13]. On the basis of 4D, $N = 4$ heterotic string theory, we provided a justification for why this bizarre structure must arise. From all of our previous investigations of 4D, $N = 4$ supergeometry [14], it can be seen that this composite $U(1)$ connection was always present (either implicitly or explicitly). It was therefore natural to conclude that since 4D, $N = 1$ heterotic strings are closely related to 4D, $N = 4$ heterotic strings, a remnant of the $N = 4$ $U(1)$ composite connection could occur in the $N = 1$ theory.

The new Green-Schwarz formulation has now completely vindicated our deductive reasoning! It has now been rigorous derived that the pure 4D, $N = 1$ supergravity limit of the heterotic string is (written in *Superspace* conventions as in [7])

4D, $N = 1$ β FFC Supergeometry

$$\begin{aligned}
[\nabla_\alpha, \nabla_\beta] &= 0 \quad , \\
[\nabla_\alpha, \bar{\nabla}_{\dot{\alpha}}] &= i\nabla_{\underline{a}} + H_{\beta\dot{\alpha}}\mathcal{M}_\alpha{}^\beta - H_{\alpha\dot{\beta}}\bar{\mathcal{M}}_{\dot{\alpha}}{}^{\dot{\beta}} \quad , \\
[\nabla_\alpha, \nabla_{\underline{b}}] &= i(\nabla_\beta H_{\gamma\dot{\beta}})\mathcal{M}_\alpha{}^\gamma - i\frac{1}{2}(\nabla_{(\alpha}H_{\beta)\dot{\gamma}})\bar{\mathcal{M}}_{\dot{\beta}}{}^{\dot{\gamma}} + i\frac{1}{2}(\nabla_{(\alpha}H_{\beta)\dot{\beta}})\mathcal{Y} \\
&\quad + iC_{\alpha\beta}[\bar{W}_{\dot{\beta}\dot{\gamma}}{}^{\dot{\delta}}\bar{\mathcal{M}}_{\dot{\delta}}{}^{\dot{\gamma}} + \frac{1}{6}\nabla^\delta H_{\delta\dot{\gamma}}\bar{\mathcal{M}}_{\dot{\beta}}{}^{\dot{\gamma}} - \frac{1}{2}\nabla^\beta H_{\beta\dot{\beta}}\mathcal{Y}] \quad , \\
[\nabla_{\underline{a}}, \nabla_{\underline{b}}] &= \{ i\frac{1}{2}C_{\alpha\beta}H^\gamma{}_{(\dot{\alpha}}\nabla_{\gamma\dot{\beta})} - \frac{1}{4}C_{\alpha\beta}[\bar{\nabla}_{(\dot{\alpha}}\nabla^\delta H_{\delta\dot{\beta})} + i2\nabla^\gamma{}_{(\dot{\alpha}}H_{\gamma\dot{\beta})}]\mathcal{Y} \\
&\quad + [C_{\dot{\alpha}\dot{\beta}}(W_{\alpha\beta}{}^\gamma - \frac{1}{6}(\bar{\nabla}^{\dot{\gamma}}H_{(\alpha\dot{\gamma}})\delta_{\beta)}{}^\gamma) - \frac{1}{2}C_{\alpha\beta}(\bar{\nabla}_{(\dot{\alpha}}H^\gamma{}_{\dot{\beta})})]\nabla_\gamma \\
&\quad - [C_{\dot{\alpha}\dot{\beta}}W_{\alpha\beta\gamma\delta} + \frac{1}{4}C_{\gamma(\alpha}(\nabla_\delta\bar{\nabla}^{\dot{\delta}}H_{|\beta)\dot{\delta}}) \\
&\quad \quad + \frac{1}{4}C_{\gamma(\alpha}(\bar{\nabla}^{\dot{\delta}}\nabla_{|\beta)}H_{\delta\dot{\delta}})]\mathcal{M}^{\gamma\delta} \\
&\quad + \frac{1}{6}C_{\gamma(\alpha}C_{\beta)\delta}[\nabla^\epsilon\bar{\nabla}^{\dot{\epsilon}}H_{\epsilon\dot{\epsilon}}]\mathcal{M}^{\gamma\delta} \\
&\quad + \frac{1}{2}C_{\alpha\beta}[\nabla_\gamma\bar{\nabla}_{(\dot{\alpha}}H_{\delta\dot{\beta})}]\mathcal{M}^{\gamma\delta} + h.c. \quad \} \quad , \\
\end{aligned} \tag{4.1}$$

$$\begin{aligned}
H_{\alpha\beta\gamma} &= H_{\alpha\beta\dot{\gamma}} = H_{\alpha\beta\gamma} = H_{\alpha\beta\underline{c}} \quad , \quad H_{\alpha\dot{\beta}\underline{c}} = i\frac{1}{2}C_{\alpha\dot{\gamma}}C_{\dot{\beta}\dot{\gamma}} = 0 \quad , \\
H_{\underline{abc}} &= 0 \quad , \quad H_{\underline{abc}} = i\frac{1}{4}[C_{\beta\gamma}C_{\dot{\alpha}(\dot{\beta}}H_{\alpha\dot{\gamma})} - C_{\dot{\beta}\dot{\gamma}}C_{\alpha(\beta}H_{\gamma)\dot{\alpha}}] \quad . \quad (4.2)
\end{aligned}$$

The superspace torsions ($T_{\underline{AB}}^{\underline{C}}$), Lorentz curvatures ($R_{\underline{AB}\gamma}^{\delta}$ and $R_{\underline{AB}\dot{\gamma}}^{\dot{\delta}}$) and \mathcal{R} -symmetry field strength ($F_{\underline{AB}}$) can be read off by noting that in general we have

$$\left[\nabla_{\underline{A}}, \nabla_{\underline{B}} \right] = T_{\underline{AB}}^{\underline{C}} \nabla_{\underline{C}} + R_{\underline{AB}\gamma}^{\delta} \mathcal{M}_{\delta}^{\gamma} + R_{\underline{AB}\dot{\gamma}}^{\dot{\delta}} \mathcal{M}_{\dot{\delta}}^{\dot{\gamma}} + F_{\underline{AB}} \mathcal{Y} \quad , \quad (4.3)$$

where $\mathcal{M}_{\delta}^{\gamma}$ and $\mathcal{M}_{\dot{\delta}}^{\dot{\gamma}}$ refer to the anti-self dual and self-dual parts, respectively, of the Lorentz generators multiplied by Pauli matrices. In the same vein, \mathcal{Y} refers to the generator of \mathcal{R} -symmetry. Equations (4.1) and (4.2) show that all of the geometrical quantities are expressed solely in terms of $H_{\underline{a}}$ (at lowest order in θ the axion field strength), $W_{\alpha\beta\gamma}$ (at lowest order in θ the gravitino field strength) and their spinorial derivatives. The most remarkable feature of (4.1) and (4.2) is the fact that they do not contain the auxiliary field multiplets ($G_{\underline{a}}$ and R)!

Similarly, the general expression of the axion multiplet field strength defined by $H_{\underline{ABC}}$ whose components for various choices of Lorentz indices is explicitly given by,

$$\begin{aligned}
H_{\alpha\beta\gamma} &\equiv \frac{1}{2}\nabla_{(\alpha|}B_{|\beta\gamma)} - \frac{1}{2}T_{(\alpha\beta|}^{\underline{E}}B_{\underline{E}|\gamma)} \quad , \\
H_{\alpha\beta\dot{\gamma}} &\equiv \nabla_{(\alpha|}B_{|\beta)\dot{\gamma}} + \nabla_{\dot{\alpha}}B_{\alpha\beta} - T_{\alpha\beta}^{\underline{E}}B_{\underline{E}\dot{\gamma}} - T_{\dot{\gamma}(\alpha|}^{\underline{E}}B_{\underline{E}|\beta)} \quad , \\
H_{\alpha\beta\underline{c}} &\equiv \nabla_{(\alpha|}B_{|\beta)\underline{c}} + \nabla_{\underline{c}}B_{\alpha\beta} - T_{\alpha\beta}^{\underline{E}}B_{\underline{E}\underline{c}} - T_{\underline{c}(\alpha|}^{\underline{E}}B_{\underline{E}|\beta)} \quad , \\
H_{\alpha\dot{\beta}\underline{c}} &\equiv \nabla_{\alpha}B_{\dot{\beta}\underline{c}} + \nabla_{\dot{\beta}}B_{\alpha\underline{c}} + \nabla_{\underline{c}}B_{\alpha\dot{\beta}} - T_{\alpha\beta}^{\underline{E}}B_{\underline{E}\underline{c}} \\
&\quad - T_{\underline{c}\alpha}^{\underline{E}}B_{\underline{E}\dot{\beta}} - T_{\underline{c}\dot{\beta}}^{\underline{E}}B_{\underline{E}\alpha} \quad , \\
H_{\alpha\underline{b}\underline{c}} &\equiv \nabla_{\alpha}B_{\underline{b}\underline{c}} - \nabla_{[\underline{b}}B_{\alpha]\underline{c}} - T_{\alpha[\underline{b}}^{\underline{E}}B_{\underline{E}]\underline{c}} - T_{\underline{b}\underline{c}}^{\underline{E}}B_{\underline{E}\alpha} \quad , \\
H_{\underline{a}\underline{b}\underline{c}} &\equiv \frac{1}{2}\nabla_{[\underline{a}}B_{|\underline{b}\underline{c}]} - \frac{1}{2}T_{[\underline{a}\underline{b}}^{\underline{E}}B_{\underline{E}]\underline{c}} \quad . \quad (4.4)
\end{aligned}$$

Here $B_{\underline{AB}}$ refers to a super 2-form whose rigid geometry was given in [15] and whose local supergeometry, implied the 4D, $N = 1$ heterotic string, can be read by comparing (4.2) with (4.4). All components of the field strength not explicitly written above may be obtained by complex conjugation.

Although the constraints in (4.1) and (4.2) are the most convenient from the view of heterotic string theory, they are by no means unique. As shown in [7], there exist field re-definitions that we call “entangling” that can be used to relate these to any specific supergeometry that contains minimal off-shell supergravity plus a tensor multiplet. Along the lines of a historic perspective, we note that the first appearance of this class of superspace geometries was within the context of 10D, N

$= 1$ β -function calculations [16]. The analogs of these constraints were found to have the consequence that the entire one-loop contribution to the β -function comes from a single graph. For this reason, the constraints have been called the β FFC (beta function favored constraint) supergeometry. The results of (4.1) and (4.2) are the direct descendants of their 10D progenitors and a major discovery of [7] was to show that the inclusion of the composite \mathcal{R} -symmetry connection allowed these to appear in 4D, $N = 1$ supergeometries.

A few words are in order as to how the work of Berkovits and Siegel rigorously leads to (4.1). As noted in [7], the standard 4D, $N = 1$ GS σ -model action actually possesses *spacetime* superconformal symmetry. As such, there is no way to distinguish what set of auxiliary fields are associated with the 4D, $N = 1$ supergravity theory coupled to the GS action. The Berkovits-Siegel action explicitly breaks the spacetime superconformal symmetry by coupling a scale compensator superfield (not the usual dilaton) in the Fradkin-Tseytlin term. It is a well known result [17] of supergravity theory that a given Poincaré supergravity theory is associated with a given choice of scale compensator. In particular, if the scale compensator is chiral, the resulting Poincaré supergravity theory must be the “old minimal” theory. The FT term in [6] can only accommodate a chiral compensator and thus the pure supergravity sector of the 4D, $N = 1$ heterotic string is the old minimal theory.

One final noteworthy consequence of the now rigorous derivation of the pure 4D, $N = 1$ supergravity limit has to do with results that have previously been accepted as facts about the phenomenological relevant form of the low-energy effective action. In a large part of the literature on string-inspired model building, the axion is represented as a pseudo-scalar that is part of a chiral superfield. The new results suggest that with the requirement of manifest 4D, $N = 1$ supersymmetry, the axion must necessarily be represented as a 2-form whose field strength couples to matter as the gauge-field for \mathcal{R} -symmetry. At a minimum, issues that until now have been considered settled must be re-examined.

5 The κ -symmetry Transformation: Birth, Death and Resurrection

One of the fascinating points regarding the new covariant formulation of 4D, $N = 1$ superstrings is the fate of κ -symmetry. This symmetry originally was found in the superparticle action [18] and was later interpreted to be related to twistor

transformations [19]. In a prior attempt to use Batalin-Vilkovisky quantization [20], it was found that κ -symmetry was the primary villain that prevented a successful quantization [21]. On the otherhand, within our prior investigation of 4D, $N = 1$ GS actions, κ -symmetry was an important tool that was responsible for the correct deduction of the form of the pure 4D, $N = 1$ supergravity limit of heterotic string theory. So it is useful to revisit this issue in light of the new formulation.

Foremost, we observe that there is no invariance in the complete 4D, $N = 1$ theory formulated by Berkovits, that corresponds to κ -symmetry. Never the less, a part of the total action does realize κ -symmetry, i.e. a sector of the total action is κ -symmetry invariant. Some years ago, we suggested that a completely consistent and manifestly supersymmetric formulation of 4D superstrings would possess this property [22] and named those terms of the complete action with this property, the “kernel” of the theory. Let us be a bit more explicit, if one writes out the complete action of [6], then one finds it can be written (with an appropriate change of notation) as the terms included in (5.2) below plus many other terms (e.g. the FT-term. etc.). Only (5.2) constitutes the kernel.

The most general kernel of a 4D GS σ -model can be constructed as follows. Introduce world-sheet fields (that must ultimately be embedded into a larger theory) $\mathcal{Z}^{\underline{M}} \equiv (\Theta^{\mu i}, \Theta^{\mu i'}, \bar{\Theta}^{\dot{\mu}}_i, \bar{\Theta}^{\dot{\mu}}_{i'}, X^{\mu\dot{\mu}})$. The Grassmann coordinates $(\Theta^{\mu i}(\sigma, \tau)$ and $\Theta^{\mu i'}(\sigma, \tau))$ are introduced in the form of two-component spinors which carry additional “isospin” indices i and i' (where $i = 1, \dots, N_L$ and $i' = 1, \dots, N_R$ for some integers N_L and N_R). We introduce the symbol $\hat{H}_{\underline{A}\underline{B}\underline{C}}$ defined by

$$\hat{H}_{\underline{A}\underline{B}\underline{C}} = i\frac{1}{2}C_{\alpha\gamma}C_{\dot{\beta}\dot{\gamma}} \left\{ \begin{array}{ll} \delta_i^j & : \text{if } \underline{A} = \alpha i, \underline{B} = \dot{\beta} j, \underline{C} = \gamma\dot{\gamma} \\ & \text{or any even permutation,} \\ -\delta_i^j & : \text{for any odd permutation,} \\ -\delta_{i'}^{j'} & : \text{if } \underline{A} = \alpha i', \underline{B} = \dot{\beta} j', \underline{C} = \gamma\dot{\gamma} \\ & \text{or any even permutation,} \\ \delta_{i'}^{j'} & : \text{for any odd permutation,} \\ 0 & : \text{otherwise.} \end{array} \right\} . \quad (5.1)$$

and using this symbol the kernel takes a universal form

$$\begin{aligned} \mathcal{S}_{GS}^{Kernel} &= \int d^2\sigma e^{-1} \left[-\Pi_{\#}^{\underline{a}}\Pi_{\#}^{\underline{b}}\eta_{\underline{a}\underline{b}} + \int_0^1 dy \hat{\Pi}_y^{\underline{A}}\Pi_{\#}^{\underline{B}}\Pi_{\#}^{\underline{C}}\hat{H}_{\underline{A}\underline{B}\underline{C}} \right] , \\ \hat{\mathcal{Z}}^{\underline{M}} &\equiv \mathcal{Z}^{\underline{M}}(\sigma, \tau, y) \quad , \quad \Pi_y^{\underline{A}} \equiv (\partial_y \mathcal{Z}^{\underline{M}})E_{\underline{M}}^{\underline{A}}(\hat{\mathcal{Z}}) \quad , \quad \hat{H}_{\underline{A}\underline{B}\underline{C}} \equiv \hat{H}_{\underline{A}\underline{B}\underline{C}}(\hat{\mathcal{Z}}) . \end{aligned} \quad (5.2)$$

where we have used the Vainberg construction to express the action in terms of the field strength of the 2-form [23].

As shown in [21], for arbitrary values of N_L and N_R it is possible to define κ -symmetry variations that leave the kernel invariant. The reason this is interesting is because the number of space-time supersymmetries, N is the sum $N_L + N_R$. We pick $N_L = 1$, $N_R = 0$ to define the 4D, $N = 1$ theory. For $N = 2$, there are two choices, either $N_L = 2$, $N_R = 0$ or $N_L = 1$, $N_R = 1$. Precisely, these cases can be seen in the recent work of [6]. Apparently, the first of the two choices corresponds to the heterotic compactification and the second to a type-II compactification. The fact that this remnant of structures found through the use of κ -symmetry arguments survives into the full theory provides us with a second example showing that though κ -symmetry is broken, its use can still lead to useful insights and questions. Of course we expect these results to generalize to higher values of N_L and N_R . For example, for $N = 3$, there are two different (N_L, N_R) possibilities; (3,0) and (2,1). The first is clearly a heterotic compactification. The second, however, is quite mysterious. All type-II compactifications are expected to have $N_L = N_R = \frac{1}{2}N$. Considering the case of $N = 4$ there are possibilities; (4,0), (3,1) and (2,2). Finally for $N = 8$, we see the possibilities, (8,0), (7,1), (6,2), (5,3) and (4,4). Once again we identify the first with the heterotic case and the last with the type-II case. The others are again mysterious. Should the intermediate cases prove to lead to consistent superstrings, they would be examples of 4D superstrings that do not have their origins in 10D.

6 Solving the 4D, $N = 1$ Conformal Symmetry Problem and Beyond

In a previous work [7], we found a puzzling situation. For the usual 4D, $N = 1$ GS non-linear σ model, the condition of κ -symmetry invariance implied that the 4D, $N = 1$ superspace could describe any space-time superconformal background. At the time of our discovery of this fact, we pointed out that this situation clearly must be resolved in order to have a 4D, $N = 1$ heterotic string whose point particle limit yielded Poincaré as opposed to conformal supergravity theory.

Once again, the work by Berkovits and Siegel provides a simple resolution to this problem. By introducing the covariant scale compensator as the coefficient of the world sheet curvature in the Fradkin-Tseytlin term, the apparent superconformal symmetry of the 4D, $N = 1$ GS non-linear σ -model is easily broken in precisely the same manner that conformal symmetry is broken in superfield supergravity theories. This suggests that 4D, $N = 1$ superstring theory (and quite likely all superstring theories) follow the paradigm of the superfield supergravity formulation involving

pre-potentials. We had long conjectured that this would be the case [4].

A final remaining challenge we must undertake is in the realm of compactification. Although the work of Berkovits and Siegel only contains explicit results for a Calabi-Yau compactified sector, there are good reasons to derive explicit results for other types of compactifications (some of which cannot even be interpreted as higher D theories). Phenomenologically, there is no guarantee that the simplest string-extended standard model is a member of the class of Calabi-Yau compactifications. As we have noted in our previous σ -model investigation [24] of Calabi-Yau compactification, CY σ -models seem to be only a special case of a more general class of models. In fact, it appears that our work within the NSR formulation of the σ -model has the interesting feature that its Calabi-Yau sector is exactly the same as that of the new GS formulation [25]. We therefore believe that this property will be true of any consistently formulated NSR compactification sector! The most general members of the σ -model compactification sectors we have found are the Lefton-Righton Thirring Models (LRTM) [4, 26]. So in a future work we will investigate the implication of the new GS formulation within the (2,0) LRTM class.

Acknowledgment;

The author wishes to thank N. Berkovits and W. Siegel for enlightening conversations about the new manifestly supersymmetric string theory formulation.

References

- [1] S. J. Gates, Jr. and H. Nishino, *Class. Quant. Grav.* **3** (1986) 391.
- [2] R. Brooks, S. J. Gates, Jr. and F. Muhammad, *Class. Quant. Grav.* **3** (1986) 745; S.J. Gates, Jr. and P. Majumdar, *Mod. Phys. Lett.* **A4** (1988) 339; *idem.* *Phys. Lett.* **248B**, (1992) 71.
- [3] J. Kowalski-Glikman, *Phys. Lett.* **180B**, (1986) 359; J. Kowalski-Glikman and J. W. van Holten, *Nucl. Phys.* **B283** (1987) 305; J.M.L. Fisch, *Phys. Lett.* **219B**, (1992) 71; J. Kowalski-Glikman, J. W. van Holten, S. Aoyama, and J. Lukierski, *Phys. Lett.* **201B**, (1988) 487; D. P. Sorokin, V. I. Tkach, D. Volkov and A. A. Zheltukin, *Phys. Lett.* **216B** (1989) 302; *idem.* *Mod. Phys. Lett.* **A4** (1989) 901.
- [4] S. J. Gates, Jr., in the *Proceedings of the XXV Winter School of Theoretical Physics* in Karpacz, Poland Birkhauser-Verlag Press (Feb., 1989) 169.
- [5] N. Berkovits, *Nucl. Phys.* **B395** (1993) 77; *idem.* *Nucl. Phys.* **B420** (1994) 332; *Nucl. Phys.* **B431** (1994) 258.
- [6] N. Berkovits and W. Siegel, SUNY Stony Brook ITP preprint ITP-SB-95-41, (Oct., 1995) hep-th 9510106.
- [7] S. J. Gates, Jr., P. Majumdar, R. Oerter and A. E. van de Ven, *Phys. Lett.* **214B**, (1988) 26; *idem.* *Nucl. Phys.* **B319** (1989) 291.
- [8] R. Brooks, S. J. Gates, Jr. and F. Muhammad, *Nucl. Phys.* **B268** (1986) 599.
- [9] S. J. Gates, Jr., Univ. of Maryland Preprint UMDEPP-96-19, (Aug., 1995) hep-th 9508153.
- [10] C.M. Hull, B. Spence, *Nucl. Phys.* **B345** (1990) 493 C.M. Hull, *Mod. Phys. Lett.* **A5** (1990) 1793; S.J. Gates, Jr. and S. V. Ketov, *Phys. Lett.* **271B** (1990) 355.
- [11] D. P. Sorokin, V. I. Tkach, D. Volkov and A. A. Zheltukin, *Phys. Lett.* **216B** (1989) 302.
- [12] S. Cecotti, S. Ferrara and L. Girardello, *Phys. Lett.* **198B** (1987) 336; B. Ovrut, in the *Proceedings of the Sainte-Adele 1988 Group Theoretical Methods in Physics* 733; *idem.* *Phys. Lett.* **205B** (1988) 455; *idem.* *Phys. Lett.* **214B** (1988) 177; *Nucl. Phys.* **B321** (1989) 163.
- [13] W. Siegel, *Phys. Lett.* **211B** (1988) 55. (1989) 291.

- [14] S.J. Gates, Jr., Nucl. Phys. **B213** (1983) 409; S.J. Gates, Jr. and B. Zwiebach, Phys. Lett. **123B** (1983) 200; idem. Nucl. Phys. **B238** (1984) 99; S. J. Gates, Jr. and J. Durachta, Mod. Phys. Lett. **A21** (1989) 2007.
- [15] S.J. Gates, Jr., Nucl. Phys. **B184** (1981) 381.
- [16] H. Nishino, M.T. Grisaru and D. Zanon, Phys. Lett. **206B** (1988) 625; idem. Nucl. Phys. **B314** 363; idem. in the *Proceedings of the Superstring Workshop at the University of Maryland* World Scientific Press (1988) 349.
- [17] S.J. Gates, Jr., in the *Proceedings of the Supergravity Workshop* at Stony Brook, 1979, eds. D. Freedman and P. van Nieuwenhuizen, North-Holland, 1979, Amsterdam; S. J. Gates, Jr., K.S. Stelle and P.C. West, Nucl. Phys. **B169** (1980) 347, B. Bergshoeff, M. de Roo, J. W. van Holten, B. de Wit and A. Van Proeyen, in the *Proceedings of the Supergravity Workshop* Cambridge, 1979, eds. M. Roček and S. Hawking, Cambridge Univ. Press, 1980, Cambridge; E. Bergshoeff, M. de Roo, B. de Wit, Nucl. Phys. **B182** (1981) 173; idem. B. de Wit, Presented at 2nd Europhysics Study Conf. on Unification of Fundamental Interactions, Erice, Italy, Oct 6-14, 1981. .
- [18] W. Siegel, Phys. Lett. **128B** (1983) 397.
- [19] E. Witten, Nucl. Phys. **B266** (1986) 245.
- [20] S. J. Gates, Jr., M.T. Grisaru, U. Lindström, M. Roček, W. Siegel, P. van Nieuwenhuizen and A. E. van de Ven, Phys. Lett. **225B** (1989) 44.
- [21] M. Henneaux and J.M.L. Fisch, Santiago, Centro Estudios Científicos preprint, ULB-TH2/89-04, (June, 1989).
- [22] S. J. Gates, Jr. in the *Proceedings of the Superstring and Particle Theory Conference in Tuscaloosa, AL* (1989), World Scientific Press (1990) 57.
- [23] M. Henneaux and L. Mezincescu, Phys. Lett. **152B**, (1985) 340.
- [24] S. J. Gates, Jr. and Tristan Hübsch, Phys. Lett. **226B** (1989) 100; idem., Nucl. Phys. **B343** (1990) 741.
- [25] N. Berkovits, private communication.
- [26] D. Depireux, S. J. Gates, Jr. and Q-Han Park, Phys. Lett. **224B** (1989) 364; S. Bellucci, D. Depireux and S. J. Gates, Jr. Phys. Lett. **232B** (1989) 67.